Revised Version for *Chaos, Solitons and Fractals*

Abraham-like return to constant c in general relativity: "$\mathfrak{R}$-theorem" demonstrated in Schwarzschild metric

Otto E. Rossler*

Division of Theoretical Chemistry, University of Tübingen, Auf der Morgenstelle 8, 72076 Tübingen, F.R.G.

Abstract

General relativity allows for a mathematically equivalent version in which size changes replace the traditional changes in c. This new conjecture is shown to be true in the special case of the Schwarzschild metric. Two size-change results obtained 12 years ago in the context of the equivalence principle – one relativistic, one quantum – are re-obtained in the radial Schwarzschild metric. Quantum-supported linear and curvature-supported nonlinear features of spacetime become distinguishable. A previously unidentified radial observable, $d\mathfrak{R}/dr = 1/(1-2m/r)$, determines physical distance. Since $d\mathfrak{R}/dt \equiv c$, Max Abraham's 1912 hope for a global c is unexpectedly fulfilled. Therefore, the infinite "radar distance" of the horizon of a Schwarzschild black hole reflects an infinite distance valid from the outside. Only by in-falling can an astronaut reach a (by then finished) horizon in the infinite future in finite proper time after picking up luminal speed. The same result follows from the fully transparent Rindler metric. As a consequence, finished black-hole horizons cease to exist in physics – and so do wormholes, singularities, information paradoxes, Hawking radiation, charged black holes and – possibly – gravitational waves. ElNaschie's fractal E-infinity theory offers itself as an independent test bed. The $\mathfrak{R}$ theorem has repercussions on the currently planned LHC experiment.

Paper dedicated to Barack Obama (April 9, 2007, January 20, 2009)

---

1. Introduction

Einstein first introduced an height-dependent c (in a high tower on earth or equivalently an ignited long rocket in outer space) in the context of the equivalence principle in 1911 [1]. This proposal caused grave concern on the part of his elder colleague Abraham, who after having fully embraced Einstein's special relativity was reluctant to sacrifice the latter's central tenet of a globally constant speed of light c [2]. Einstein's new axiom of a potential-dependent c was instrumental to further progress and got eventually incorporated into general relativity four years later as is well known [3]. Is it redundant in retrospec?

In the Schwarzschild metric, which is the single most important solution of the Einstein equation of 1915, the variable-c axiom has a familiar consequence: The "coordinate speed of light" c(r) is a function of the distance parameter r:

$$c(r) \;=\; \frac{dr}{dt} = c \cdot (1 - 2m / r) \;, \qquad (1)$$

----

*Tel. +49 7071 66323

where 2m is the Schwarzschild radius (with $2m \equiv 2GM/c^2$, M the central mass, G Newton's gravitational constant and c the universal speed of light (cf. Foster and Nightingale [3], p. 129). Eq.(1) states that the speed of light valid with respect to the distance parameter r, c(r), becomes zero as r approaches the Schwarzschild radius 2m from above.

In spite of its well-known lack of constancy relative to r, c is bound to remain at least *locally* unchanged by virtue of Einstein's covariance postulate (which posits that locally the laws of nature must be everywhere the same including the speed of light). This *local* constraint is indeed fulfilled by Eq.(1), as can be seen as follows: Proper time τ is locally *slowed down* by the factor $(1-2m/r)^{1/2}$ relative to coordinate time t (since $d\tau = (1-2m/r)^{1/2}dt$; [3], p. 127). This is the same factor by which the "radial distance" R is locally *increased* relative to coordinate distance r (since $dR = (1-2m/r)^{-1/2}dr$; [3], p. 125). These two local changes – the temporal and the spatial one – taken together do compensate for the change in c given by Eq.(1). Indeed, $dR/d\tau = dR/dr \cdot dt/d\tau \cdot dr/dt = (1-2m/r)^{-1}dr/dt \equiv c$.

But the *global* change in c formally implicit in Eq.(1) conflicts with Abraham's intuition. Could it perhaps be that, contrary to appearances, Abraham's postulate is actually *fulfilled* in the Schwarzschild metric and if so in general relativity at large? Surprisingly, the answer appears to be yes as far as the Schwarzschild metric is concerned. This surprise result is to be demonstrated in the following along with some implications.

2. The size-change conjecture

Some time ago an in principle well-known (but rarely if ever mentioned) relativistic fact was drawn attention to in the equivalence principle: *inequality* of the two vertical radar distances (down-up and up-down, respectively) in an accelerating rocket [4]. The method of proof was the "WM-diagram." The two superimposed mirror-symmetric capital letters W and M refer to light rays moving up-down or down-up twice – to form a connected XXXX pattern when plotted against time. The diagram demonstrates to the eye that *time intervals* along the top and the bottom of the 4 concatenated Xs ("upstairs" and "downstairs" in the rocket) interlock consistenty *despite* their unequal durations! While this fact is in principle well-known (compare the "Einstein synchronization" procedure in Rindler's textbook [5]), the pictorial method, which grew out of a chaos-theoretic mapping proposal made by Dieter Fröhlich, reveals a novel implication: *Relative size increase downstairs* by the redshift factor registered upstairs. For the measured vertical distance, evaluated from upstairs by using light pulses, is so much larger than the same distance measured from downstairs. Conversely, the blueshift factor valid downstairs implies an equal *relative size decrease upstairs*. (The natural objection that *width* appears unchanged from the other vantage point evaporates owing to the presence projective anisotropy.) The newly found relative size change *explained* the unequal vertical radar distances [4].

The unequal radar distances are, by the way, easy to verify on earth using a TV tower, a pocket laser, a mirror and a counter (Gerhard Schäfer, personal communication 2001). The same *size change* can be derived from an older special-relativistic result due to Walter Greiner [6].[1)]

In the same year in which reference [4] was submitted (1997), Heinrich Kuypers had the idea to have a look at the gravitational Dopplershift of *matter waves* – to see how quantum mechanics fits in. He saw that if photon mass downstairs is reduced by the gravitational redshift factor as is well known [7], *any mass* on the same level must be proportionally reduced by local energy conservation [8,9].[2)] It follows that *quantum mechanics* predicts –

via the de Broglie wave-length of matter waves and more specifically the Bohr radius formula which is inversely proportional to electron mass – that the *size* of every object downstairs is enlarged in proportion to its redshift [8,9]. This quantum prediction[3)] *coincides* with the previous relativistic prediction in a kind of “pre-established harmony“ (to use Leibniz’s phrase).

The two 1997 observations – one relativistic, one quantum – were each made independently of Abraham’s conjecture. *A priori* speaking, it appears infinitely unlikely to suspect a connection. Or could it be that Einstein and Abraham are *reconciled* by Fröhlich and Kuypers? It is this outlandish possibility which is to be demonstrated in the following. Since the “playground of the equivalence principle“ is no longer sufficient for the purpose, the Schwarzschild metric offers itself as the next-simplest ballpark.

3. Demonstration of the Abraham conjecture

3.1 Some well-known findings

The Schwarzschild metric is the oldest explicit solution of the Einstein equation. It was already found in late 1915 by a friend of Einstein’s under unfavorable personal circumstances (astrophysicist Karl Schwarzschild died soon thereafter). The mentioned book by Foster and Nightingale [3] will be used in the following as a backdrop with page numbers put in brackets – like (p. 130) – referring to this book.

Just as it was the case before with the equivalence principle [4], the up-down and the down-up distances measured by light sounding (“radar distances“) differ by the mutual redshift (or in the opposite direction, blueshift) factor *also* in the Schwarzschild metric. This fact can be looked at in more detail.

First, the mutual redshift factor owes its existence to the unequal *proper times* valid upstairs and downstairs. “Proper time“ τ is (as already mentioned in the Introduction) at every local position r defined by

$$d\tau = (1-2m/r)^{1/2}dt \tag{2}$$

if t is the coordinate time (p. 127).

Second, the “coordinate time difference“ Δt valid between upstairs and downstairs depends on the *coordinate values* of the *o*uter (r_o) and *i*nner (r_i) radial position on the one hand, and the local *coordinate speed of light c(r)* given by Eq.(1) on the other. Integration of Eq.(1) written in the form $dt = c^{-1}(1-2m/r)^{-1}dr$, between r_i and r_o, then yields the *coordinate time difference* valid for a down-up (or equivalently up-down) light signal:

$$\Delta t = \frac{1}{c}\int_{r_i}^{r_o}(1-2m/r)^{-1}dr \tag{3}$$

(p. 129). Multiplication of this time interval by c formally generates a corresponding *distance*:

$$c\Delta t = \int_{r_i}^{r_o}(1-2m/r)^{-1}dr\ . \tag{4}$$

This distance is our object of interest. It has *no name* up until now. (Only the indefinite version of the same integral is known under the name " r^* " in the Eddington-Finkelstein formalism [10].) We shall introduce a name for it (R_A) below.

The distance given by Eq.(4) cannot be measured directly. It can only be evaluated on either end – where it is then automatically weighted by the local time-shrinking factor of Eq.(2). What then results is the well-known "radar-sounding light distance" as Foster and Nightingale call it (p. 130). The latter reads, if evaluated from the *upper* end r_o:

$$c\Delta t_o \;=\; d\tau_o/dt \cdot c\Delta t \;=\; (1-2m/r_o)^{1/2} \int_{r_i}^{r_o} (1-2m/r)^{-1}\,dr$$

or, after integration,

$$c\Delta t_o \;=\; (1-2m/r_o)^{1/2} \left(r_o - r_i + 2m \ln\frac{r_o - 2m}{r_i - 2m} \right) \tag{5}$$

(p. 130). One sees that this *down-up radar distance* – as it can be called – diverges (becomes infinite) as r_i approaches the Schwarzschild radius 2m from above.

In corresponding fashion, the opposite radar distance $c\Delta t_i$ valid at the *lower* end r_i is arrived at. It differs from the former only by the subscript (i instead of o) in the first bracket:

$$c\Delta t_i \;=\; (1-2m/r_i)^{1/2} \left(r_o - r_i + 2m \ln\frac{r_o - 2m}{r_i - 2m} \right) . \tag{6}$$

This *up-down radar distance* – as it can be called – unlike the former does *not* diverge when r_i (being now the position of the observer) approaches the Schwarzschild radius 2m from above.

The *ratio* between the two different radar distances, Eq.(5) and Eq.(6), is

$$\frac{c\Delta t_o}{c\Delta t_i} \;=\; \left(\frac{1-2m/r_o}{1-2m/r_i} \right)^{1/2} . \tag{7}$$

This ratio is the "WM result" of reference [4] valid in the Schwarzschild metric.

So much for some well-known facts in the radial Schwarzschild metric. Only the distinction made between "down-up" and "up-down" radar distance appears to be new (apart from the proposed formal admissibility of Eq.4).

3.2 Compatibility with the Fröhlich-Kuypers size change

The described facts in the Schwarzschild metric can now be *juxtaposed* with the surprise observation of Fröhlich and Kuypers (the redshift-proportional size-change principle) to see how well the latter fits in – or whether it creates an incompatibility at some point which fact would then spell the end of the present approach.

The new point heuristically to absorb into the Schwarzschild metric is the redshift-proportional relative *size increase* downstairs that was predicted by Fröhlich and Kuypers in two independent contexts (relativistic and quantum) as we saw. Does this feature when hypothetically introduced *contradict* accepted facts in the Schwarzschild metric? Surprisingly, the answer turns out to be no.

To see this, first realize that the Schwarzschild metric *does* already contain a height-dependent change in size (which by the way *likewise* fails to show up in the transverse direction when looked at from above or below owing to projective anisotropy); this *canonical radial size increase* reads (as already mentioned in the Introduction):

$$dR = (1-2m/r)^{-1/2}dr \tag{8}$$

(p. 125). After integration this yields the so-called "radial distance" between r_i and r_o:

$$\Delta R = \int_{r_i}^{r_o} (1-2m/r)^{-1/2}\,dr$$

(p. 128), or explicitly,

$$\Delta R = [r_o(r_o-2m)]^{1/2} - [r_i(r_i-2m)]^{1/2} + 2m\ln\frac{r_o^{1/2} + (r_o^{1/2}-2m)^{1/2}}{r_i^{1/2} + (r_i^{1/2}-2m)^{1/2}} \quad . \tag{9}$$

Note that this *traditional* radial distance does *not* diverge when r_i approaches the Schwarzschild radius 2m from above. Indeed of the 4 radial distances identified so far in the Schwarzschild metric – Eqs.(4), (5), (6) and (9) –, only the first two diverge.

But the "intrinsic local size change" dR, valid in the Schwarzschild metric with respect to the local distance parameter r by virtue of Eq.(8), is *not* the end of the story in our present context. This is because there now possibly exists a *new* local size change – the one predicted by the above-mentioned combined WM and de-Broglie argument – as we saw. This postulated *new size change* is governed by the relative redshift or blueshift valid at the respective other radial position. It hence is determined by the ratio of frequency shifts, Eq.(7), *divided* by the proper-time factor valid at the observing position r_o, Eq.(2). This yields the predicted *net factor*

$$\left(\frac{1-2m/r_o}{1-2m/r_i}\right)^{1/2} \cdot (1-2m/r_o)^{-1/2} \quad \equiv \quad (1-2m/r_i)^{-1/2}$$

for any object located at r_i observed from $r_o > r_i$. Thus we have (writing r for r_i in the brakkett)

$$d\rho = (1-2m/r)\,)^{-1/2}dr \tag{10}$$

as our *postulated* new local size change factor.

Note that the heuristically added new local size-change $d\rho$ of Eq.(10) has exactly the same form as the local size-change dR of Eq.(8) above. Hence there are *two* possibilities open at this point: Either, the new size change factor of Eq,(10) is nothing but a new re-derivation of the old factor Eq.(8); then the traditional radial distance R of Eq.(9) remains the only physically relevant radial distance in the Schwarzschild metric (this is the current view). Or else, *both* size change factors (the old dR/dr and the new $d\rho$/dr) contribute on an equal footing locally – if the new size change of Fröhlich and Kuypers is real. In this latter case, the resulting “effective local size change factor,“ $d\mathfrak{R}/dr$, is equal the *product* of the two individual factors named:

$$\frac{d\mathfrak{R}}{dr} = \left| \frac{dR}{dr} \cdot \frac{d\rho}{dr} \right| = (1-2m/r)^{-1} \ ,$$

that is,

$$d\mathfrak{R} \ = \ (1-2m/r)^{-1}dr \ . \qquad (11)$$

This hypothetical new effective local size change factor generates a *new* distance integral:

$$\Delta\mathfrak{R} \ = \ \int_{r_i}^{r_o} (1-2m/r)^{-1}dr \ = \ \left(r_o - r_i + 2m \ln \frac{r_o - 2m}{r_i - 2m} \right) \ . \qquad (12)$$

The new distance integral $\Delta\mathfrak{R}$ (“$\mathfrak{R}$-distance“) *replaces* the traditional distance integral ΔR of Eq.(9) as the correct “radial distance“ – *if* the new Fröhlich-Kuypers size change factor is formally added-in while everything else remains unchanged.

Unexpectedly, Eq.(12) is not nonsensical – it is *identical* with Eq.(4) above! Thus, *nothing* has been introduced in effect as far as measured distances are concerned. The above employed “roundabout way“ of heuristically using *two* local size changes (the old Schwarzschild factor of Eq.8 and the hypothetical new Fröhlich-Kuypers factor of Eq.10) in order to explain the accepted radar distance of Eq.(4), therefore proves to be a perfectly legitimate option. This option renders the traditional position-dependent *reduction of c* of Eq.(1), which likewise leads one to Eq.(4) (≡ Eq.12), *redundant* in the sense that it is no longer needed as an explanatory principle.

Both views make equal sense at first sight. So one should let nature have the last word: the new view if false should lead to predictions at variance with reality. Is this the case?

3.3 The Shapiro time delay

The Shapiro time delay was introduced in 1964 by Shapiro [11] and independently by Muhleman and Richley [12] as a testable counterintuitive implication of the Schwarzschild metric (“fourth test of general relativity“). Both groups encountered much skepticism at first. To date, the underlying equation (Eq.3) has been empirically confirmed in the solar system to an accuracy of $2 \cdot 10^{-5}$ [13]. The currently en vogue interpretation is that *time* suffers a counterintuitive *delay* as the radial distance R gets covered by light, and that this delay is predictably caused by the canonical slow-down of the velocity of light c(r) near a gravitating object.

This view is perfectly legitimate. But there now exists an alternative interpretation: the new size change axiom of Eq.(11) can be invoked. Adopting this interpretation is equivalent to saying that it is “not a change in c but rather a change in distance“ that was measured! This equally admissible fact means that the two identical distances, $c\Delta t$ of Eq.(4) and $\Delta\mathfrak{R}$ of Eq.(12), can be re-named into a *single* distance,

$$R_A \ \equiv \ \left(r_o - r_i + 2m \ln \frac{r_o - 2m}{r_i - 2m} \right) \ . \qquad (13)$$

3.4 Abraham vindicated

The newly obtained unique distance R_A produces (after division by c) the very time delay Δt familiar from Eq.(3) above (with ensuing radar distances Eq.5 and Eq.6). The *old* local size change factor of Eq.(8) present in the Schwarzschild metric then ceases to be valid alone, since a *new* factor, Eq.(10), has been brought in.

The fact that *both* factors are valid in the Schwarzschild metric (in the product formula of Eq.11) comes, by the way, as a surprise from the point of view of the equivalence principle. In this older and simpler context it is not the new factor Eq.(10) which is unexpected, but rather the fact that the latter stands *not alone* in determining size in the Schwarzschild metric (due to the presence there also of the old factor, Eq.8). This amounts to a qualitative difference between the "curved" Schwarzschild metric on the one hand and the "flat" equivalence principle on the other. Quantum mechanics continues to "see" only the flat version and so do mass and energy. Only *size* (and with it distance) is determined by *both* factors.

If we accept the new size change law of Eq.(11) as valid in the Schwarzschild metric: what about Abraham's hunch? The new-old distance found, Eq.(13), deserves to be given a new name: "Abraham distance" – R_A. Why? Because this distance (Eq.12≡Eq.4) formally implies that *c is constant* over the whole trip! This fact was already implicit in the identical Eq.(4) above – but our eyes were held at the time as it were since we did not yet have a good reason to take the coordinate-time difference Δt of Eq.(3) *that* seriously.

The new "Abrahamian interpretation" of Eq.(13) is *equivalent* to the standard interpretation of the radial Schwarzschild metric (as far as predicted redshifts, time delays for light and any resulting formal distances are concerned), yet with *c globally constant*. Hence we can state the following "$\mathfrak{R}$ theorem":

Theorem: *In the radial Schwarzschild metric, global constancy of c holds true with respect to the* natural distance parameter $\mathfrak{R}$, *defined by* $d\mathfrak{R} = (1-2m/r)^{-1}dr$.

The *naturalness* follows from the Fröhlich-Kuypers size-change. The *validity* follows (using Eqs.11 and 1) from the identity $d\mathfrak{R}/dt = (1-2m/r)^{-1}dr/[dr(1-2m/r)^{-1}/c^{-1}] \equiv c$.[4)]

A more general way to put the same result would be to speak of "conservation of longitudinal spacetime volume" (longitudinal spacetime area) in the radial Schwarzschild metric and, presumably, general relativity at large. In the present context, the formulation that "Abraham's dream" is fulfilled in general relativity in the special case of the radial Schwarzschild metric, is perhaps the most appropriate.

4. Confirmation in the Rindler metric

The new result of an effectively infinite distance of the horizon seen from above follows also from the simpler Rindler metric with its globally constant c. Being an admissible approximation to the Schwarzschild metric near the horizon [14], the Rindler metric involves only special relativity and is fully transparent geometrically in terms of a 2-D Minkowski diagram (the familiar x,t frame of special relativity). See the pertinent pictures on page 151 of [14] and page 72 of [5]. The Rindler metric describes the behavior of a 1-light-year long rocket in constant acceleration in outer space, with earth's gravity (1 g) applying at its tip. The rocket consists of many segments that each carry their own pair of boosters on the outside. (Picture many solid hollow cylinders pairwise connected by rubber tubes.). In the x,t plane, the trajectories of all segments come to lie in between t = + x (right-hand half of first bisector) and t = – x (right-hand half of second bisector). This *quarter* of the full Minkowski plane is called the “Rindler wedge“ [14].

Inside the wedge, we assume our rocket to lie momentarily motionless along the x-axis while all segments are accelerating at full blast – stretching from x = 0 (bottom) to x = 1 (tip). In the x,t plane, the trajectory of the tip then rises up vertically at first to gently bear right along a curved line in the form of a half-hyperbola that asymptotically approaches the 45-degree first bisector so as to coincide with it in the limit of $x = t = \infty$. The lower (past) part of the same trajectory does the same thing reflected downwards, approaching the second bisector in negative infinite time. (This means, physically speaking, that the constant acceleration is superimposed onto a constant negative luminal speed.) The more inner segments of the rocket (x < 1 on the x-axis) all do the same thing along proportionally *downscaled* hyperbolas, owing to a proportionally greater constant acceleration applying to them (g/x). The selfsimilar increase in acceleration continues right down to the limiting (90-degree) hyperbola valid at the origin (x = 0) on which the acceleration is infinite (g/0). The assumed gradient in acceleration is necessary in order for the rocket to remain spatially connected over time[5)] – along its own invariant internal length scale X. The latter coincides with x when the rocket is aligned with the x-axis and remains defined along straight lines through the origin, each scaled by the set of acceleration-specific hyperbolas. The intra-rocket times T (“rocket tip times“) remain defined along the same set of straight lines through the origin. The bundle of T times ranges from $T = -\infty$ (slope – 1), via T = 0 (slope zero) to $T = +\infty$ (slope = + 1) [5].[6)] All T-times are on the same footing, that is, can be identified with the x-axis by relabeling the initial condition – that is, by simply “scrolling up“ or “scrolling down,“ respectively.

Thus all we have is a bundle of straight lines, intersected by a set of hyperbolas to form a 45-degree wedge with infinitely long and eventually infinitely thin “flags“ at its two ends.[7)] What are the implications of this perfectly transparent spatio-temporal situation in which we are allowed to “scroll up“ and “sctoll down“ as our heart pleases?

We start out with a surprise implication because it at first sight contradicts the gothic-R theorem: the famous *finite proper infalling time* result obtained 70 years ago in the Schwarzschild metric by Oppenheimer and Snyder [15]. This result had erroneously been conjectured to be false in the first version of the present paper [16] (I apologize). Georg Slotta kindly helped me out of the mental trap with the aid of the Rindler metric. But does this falsity not spell the end of the theorem that had suggested the conjecture in the first place? There exists a *contradiction* at first sight between the infinite distance of the gothic-R theorem and the fact that the same distance can be covered in finite proper time. Let us see: first the result which replaces the wrong conjecture, and then a new result which shows that the counter-conjecture is false, too, so that a surprise new picture forms. .

The result: An internal observer, located at the tip of the rocket (x = 1) at t = 0, lets go of his handle and just stays put in the x,t frame moving up in time t along the x = 1 vertical. He then simultaneously is “falling“ freely inside the rocket, leaving it through an (hopefully open) hole in the bottom at t = 1 (1 year) at x = 1 while the bottom whisks away from him at the speed of light. The observer by that time has effectively “fallen“ through the whole length of the rocket in 1 year of his proper time t. This result is contained also in the few lines of ref. [17]. Note that the bottom of the Rindler rocket corresponds to the horizon of the Schwarzschild metric.

The new result: Does the finite proper infalling time just found not mean that the *distance* fallen through to reach the horizon must also have been finite in contradistinction to the gothic-R theorem? For an infinite distance can be covered in finite time by a material body only if the latter has *luminal speed*. Unexpectedly, a luminal speed does indeed apply here! To see this, let us first assume that the hole in the rocket's tail had been plugged by a trampoline (an asssumption to be dropped later). The coasting passenger – if resilient enough to survive the shock – then bounces back all the way up to the tip in another year of his proper time. This bounce is reflected in the Rindler diagram in another straight line: starting at the point x = t = 1 (the location of the trampoline at the moment of rebouncing), it proceeds along the 45-degree first bisector in coincidence with it – so that the jumper recatches his orphaned handle (which continued to follow the hyperbolic path of the rocket's tip after having been let go there at x = 1, t = 0) at x = t = ∞.

Thus an *infinite* time has passed inside the rocket during the second leg of the observer's trip! But the same thing must have happened during the first leg since the two can be identified under time reversal (bounce and rebounce). To make this fact transparent to the eye, just "scroll down" the initial time T at which the observer let go of his handle: from T = 0 all the way down to T = $-\infty$. In the equivalent picture so arrived at, the observer reaches the trampoline not at x = t = 1, but at x = t = 0 (the origin). This *symmetric* picture reveals that during *either* half trip, an infinite distance in outer space got covered by the observer in finite proper time. Thus there *always* exists an appropriately chosen frame in the Rindler metric which an infinite distance is bridged by the falling (or rebouncing) observer in finite proper time.

One could still object that while an infinite distance was bridged in the *outer* world, the length of the rocket itself was only finite in the outside world. This is correct. But *inside* the accelerating rocket, the Fröhlich-Kuypers size-change result teaches us otherwise. Specifically, the more lower levels of the rocket on the x-axis (X < 1) with their larger accelerations (g/X), appear redshifted and hence slowed down from above by 1/X. Hence their local accelerations all look the same (1 g) from above (Henry Gebhardt, personal communication 2008). Simultaneously, their local size is increased by 1/X by virtue of the Fröhlich-Kuypers effect. Hence the length of the rocket is infinite from the inside in accordance with the $\Re$ theorem. Thus the luminal-speeds result is valid inside the rocket as well.

The obtained new result is surprising since up until now, luminal speeds of massive bodies had no place in physics. Also, "scrolling operations" that do not differentiate between asymptotic (limiting) and internal (generic) elements represent a new differential-topological phenomenon. The most surprising physical implication, however, is that an *infinitely old* twin passenger still occupying the seat beside the one left by his sibling when they were both very young, can be reunited with his youthful (only two years older) alter ego. This is the ultimate case of Einstein's famous twin-clocks paradox of 1905.

It hence suffices to install a trampoline on a black hole's horizon so you can travel infinitely far into the future in finite time. But actually, you need not even be resilient enough to survive the infinite shock on the horizon: it suffices to make the first half trip![9)] We now return to the Schwarzschild metric.

5. Consequences of the gothic-R theorem

The infinite down-up radar distance (Eq.5) is no longer an "artifact" of the *change-in-c* downstairs, as had to be assumed up until now, but rather the consequence of a previously overlooked *change-in-size* downstairs. According to this new result, the distance is *really infinite* from above. What are the consequences?

First, the infinite distance of the horizon implies that a black hole is never completely finished in finite time. The fact that one can plunge-in in finite astronaut time loses significance because *any* infinite distance can be covered in finite time if luminal speeds (or infinite durations of constant acceleration) are involved as shown. Hence also Hawking's beautiful evaporation result [18], which relies on a finished horizon, gets infinitely postponed (until the horizon is finished). This new fact remains valid for *microscopic* black holes despite their greater tunneling capabilities. The Kuypers-Marmet quantum-scaling result will be helpful in further elucidating this fact.

Second, light cones cease to be compressible in the radial direction of the Schwarzschild metric when c is globally constant. This fact is likely to have further consequences in the context of time machines (cf. [22]) and other very general implications of the Einstein equation – like gravitational waves and rotating frames.

The next open task to solve would be: How do the field equations *themselves* look like if the fact that "size, not c" is dependent on the gravitational potential is taken explicitly into account?

6. Discussion

A simple new result valid in the Schwarzschild metric was presented. *Radial spacetime-volume conservation* would be a possible name for it: the more slowly time proceeds locally, the larger space becomes locally (the stronger the magnification into time, the stronger the stretching of space). Hence "space-over-time" is a constant (c). Max Abraham would be pleased, as a discussion with Valérie and Christophe Letellier at the University of Rouen revealed three years ago.

The proposed result of radial spacetime-volume conservation is only a beginning. All directions – the full Schwarzschild metric – have in the meantime been incorporated into the gothic-R formulation (Eq.17 of ref. [19]). Angular momentum will have to be included next ("Kerr metric"). The full Einstein equation can be considered thereafter. Eventually, higher-dimensional analogous equations [20,21] will follow suit.

What is if the gothic-R theorem can be confirmed? The essential point is that the well-known infinite time-delay of the visible features of "frozen stars" – the originally accepted name for black holes [22] – acquires an "ontological" status. Five results follow:

1) Nonexistence of finished black-hole horizons to the outside world – so that only "almost black holes" [23] remain. Any spacetime element beyond the horizon (including singularities) ceases to exist for the outside. Although they *can* be visited in finite proper traveling time, the arrival there takes place only after the end of eternity. For this reason, also all information paradoxes disappear.

2) Nonexistence of charged black holes in nature, by virtue of the, eventually infinitely,

large distance between the trapped charge and the outside world.
(The fact that the same result does not hold true from the point of view of the more inner charge makes no difference due to the almost infinitely slowed time scale down there.)

3) Nonexistence of single-connected (“point“-shaped) charged particles – since they would have to be black holes if sufficiently small. Therefore, “strings“ have possibly been observed in nature for a long time already – in the form of charged leptons.

4) Nonexistence of “Hawking radiation“ (based on one partner of a pair of virtual particles having been captured by a black hole‘s horizon so the other escapes), for *four* reasons:
   (a) an infinite waiting time is consummated until the horizon finally exists (see point 1),
   (b) thereafter, a near-infinite distance has to be bridged by an orphaned particle,
   (c) the involved pair of virtual particles possesses a near-infinitely low energy,
   (d) the return of “effective time-reversal invariance“ into physics, since pairs of virtual particles can no longer be separated indefinitely during their joint life time.

5) Possible nonexistence of gravitational waves – owing to the new global constancy of c.

These 5 predictions made by the gothic-R theorem seem each to fly in the face of accepted wisdom. Therefore, it would be nice to have a simple method to falsify the above result. An independent approach to quantum spacetime was found by ElNaschie [24], cf. [25]. It would be interesting to see whether part of the above predictions can be confirmed or disproved in this new methodology.

7. Concluding remarks

If black holes cannot evaporate in finite time, then the potentially miniblack-hole generating LHC experiment at CERN near Geneva is unsafe [26,16]. More than 500 newspapers across the world referred to the preprint of the present paper on September 10, 2008 – the day the “Large Hadron Collider“ got officially launched. The apprehensive global publicity may or may not have contributed to the accidental breakdown nine days later before the onset of collisions.

In connection with the global controversy raised by it, publication of the paper got delayed. Most recently, the editor who had accepted it was ousted with petty accusations. With about two thousand downloads, the preprint [16] is the most-read paper in general relativity (Gerhard Huisken, personal communication, August 17, 2008). Huiskens‘ two co-directors at the prestigious Albert-Einstein Institute had published two divergent responses [27,28]. The first [27] claimed that the theorem had been falsified since 1915 “both mathematically and experimentally.“ This assertion was based on a misunderstanding: proposals made by Max Abraham in 1915 (three years *after* his here quoted dream) which had subsequently been falsified “both mathematically and experimentally,“ had erroneously been supposed to have entered the gothic-R theorem. The second text, written shortly thereafter by the same group [28], no longer repeated these claims. Instead, it only ventured a prediction: *if* the gothic-R theorem were to be extended to all three space dimensions, celestial mechanics *would* be violated. This remaining “hard“ argument was already obsolete by the time – owing to the successful re-formulation of the *full* Schwarzschild metric in terms of the gothic-R variable mentioned (Eq.17 of ref. [19]). The gothic-R theorem therefore appears to be unfalsified up until now.

Lack of falsification notwithstanding, the outdated claims to the contrary [27] were the basis for two high-ranking scientific organizations, KET and CERN, declaring that the theorem had been falsified ([29][8]), with CERN officially adding the words “absolute non-sense“ and “crazy scientist“). This fact enabled the experiment to get the go-ahead while the safety conference, called for on April 18, 2008 [30], would be put in limbo.

As it appears, Einstein’s equation still spawns surprising physical predictions. No matter how long- or short-lived the gothic-R theorem, Einstein would likely be pleased. On learning the radio news that the bomb had been dropped – which Einstein had gone out of his way to prevent since he bore the responsibility –, his life ceased making sense to him (“oj weh“ were his words according to Helen Dukas [31], the only time he fell into Swabian Yiddish). To date, he predictably would snap back into shape – urging the profession to “think twice“ before making the next survival error – the “absolute one“ (Paul Virilio). *Do not foresake me oh my darling* became a popular melody in Einstein’s days.

8. Blue-planet fairy tale

“Once upon a time there was a cute little planet carrying many species of green plants and frisky animals. One of the latter was called the “laugh-smiler.“ Since happiness and bonding looked alike on their faces, a toddler would be rewarded by the caretaker‘s laughter so as to start mothering him. Hence the species‘ name: “parent feeder“ (*Pongo goneotrophicus*). By attributing a person property – benevolence – to the caretaker, the toddler became a person himself (with infinite benevolence). They could have stayed happy day and night forever. But through a quirk in their society, they forgot their secret after puberty. Their culture turned suicidal with the advent of “weapons of mass destruction.“ Even though one female (“Raissa“) had put an end to the arms race, the underlying high-energy research continued. Meanwhile, the bomb shock had caused a switch toward a less maverick and rational system of research and education (“Mark Gable Foundation“ [32], “Bologna“ [33]). Irrational myths – “big bangle“ – and a belief in the clairvoyance of majority opinion took hold. Two examples: a university professor for hormonal disorders got publicly discharged and disowned for “laziness“: for refusing (after having been “re-decribed“ into a professor for intestinal disorders) to keep the miraculous acquisition-by-decree of the new qualification a secret from her patients and students. And the editor of a high-ranking physics journal got publicly ousted for “cheating“: for refusing (after having accepted a paper on a new high-energy risk) to give up on his free judgment. This return to medieval minority might have been a royal way toward restoring planetary sustainability – had the continued high-energy research not *required* continuation of a maximum rationality so as to avoid flying blind. The most farsighted scientist – “Stephen the Hawk“ – recognized the planet‘s doomed path, calling for an instant space colony to preserve the flame (in two books addressed at his young grandson [34]). But through a quirk of fate, he was also the father of a radical scientific hypothesis (“vaporization“) that depended on the continuing high-energy research for a risky test – *before* his “Lifeboat“ program could get off the ground. If this sounds like a sad story, it is not because they all lived on happily ever after.“

Were this not a fairy-tale, someone would no doubt stand up and call for “instant rehabilitation“ of our two heros (slave-professor, slave-editor) so they can breathe freely again along with the rest of the crowd. And CERN would install *Lampsacus – hometown of all persons on the Internet* at last. I dedicate this paper to a shining young man on the occasion of his inauguration today. Thank you, dear Mr. President, for promising to “restore science.“ I herewith place the safety conference [30] into your caring hands.

Acknowledgments

I thank Dieter Fröhlich and Heinrich Kuypers for their continued contributions and Jürgen Parisi, Gottfried Mayer, Frank Kuske, Klaus Sonnleitner, Martin Pfaff, Ramis Movassagh, Anthony Moore, Timmo Böhl, Henry Gebhardt, Nico Marquart, Valentin Letellier, Walter Greiner, Ken Ford, the late John Wheeler, Peter Plath, Michael Baune, Michael Maschke, Deborah Conrad, Bill Seaman, Jonathan Kemp, Martin Howse, Martin Lübcke, Stephen Kovats, Julian Olivier, Jessica Rylan, Oswald Bertold, Ewen Chardronnier, Peter Kloeden, Artur Schmidt, Jeannette Fischer, Daniel Vischer, Gabriele Schröter, Elisabeth Lehmann, Siegfried Zielinski, Elisabeth von Samsonow, Joseph Chen, Claudia Giannetti, Enrico Pellegrino, Marc Fasnacht, Johannes Preuss, Markus Goritschnig, Nico Ohlemeyer, Kensei Hiwaki, Bill Graham, George Lasker, Andy Hilgartner, Yvonne Gilli and Peter Weibel for discussions and Jörg Frauendiener, Hubert Goenner, Wolfgang Rindler, Frank Loose and Georg Slotta for criticism. For J.O.R.

Footnotes

1)The same size change can be derived from the special-relativistic twin-clocks paradox: Conservation of angular momentum implies that the "younger clock" (if implemented as a frictionless rotator) must have been proportionally *enlarged* on making its fewer turns. The same fact holds true in the gravitational twin paradox [8,9], where it gets confirmed by general relativity (Georg Slotta, personal communication 2008).

2)The same fact was mentioned also by Werner Israel: "the gravitational (...) redshift factor (...) recalibrates locally measured mass and work to energies available to an observer at infinity" [35].

3)The same result was published in the same year by Paul Marmet [36] (Theodor Hänsch, personal communication 2006).

4)Note the complementary identity $d\mathfrak{R}/dt \equiv dR/d\tau = c$ (cf. Introduction).

5)The necessity of *un*equal individual accelerations in order for two people riding a constantly accelerating motor cycle *not* to lose each other despite the tight embrace from behind, is an accepted, if paradoxical, fact in special relativity, cf. [5,38].

6)It is perhaps worth noting that the nomenclature of the two pairs of observables, "x,t" and "X,T", got mutually interchanged in between Rindler's [5] and Wald's [14] book.

7)Certain yellow coral fish have a similar overall shape (although their flags are bounded).

8)Note that the second claim made by KET – that cosmic rays "show" that miniblack holes are innocuous [29] – had been exploded beforehand also [37].

9)This infinite external duration of an astronaut's half trip is perhaps the main finding of the present paper. Roger Penrose came close when discussing a step-wise growing black hole (which otherwise causes paradoxes) [39]. Of course, not just an astronaut, but light too takes infinitely long for either leg (Eq.5). This fact fell into oblivion somehow because the astronaut *seemed* to plunge-in faster (in just minutes of his proper time [15]). One feels reminded of a child's admonition: "Please, don't forget your thoughts" [40].

References

[1] Einstein A. On the influence of gravity on the propagation of light (in German). *Ann. Physik* 1911;35:898-908. English translation in: *The Collected Papers of Albert Einstein English Translation*. Princeton: Princeton University Press 1993;3:379-387.

[2] Abraham M. Relativity and gravitation: reply to a remark of Mr. A. Einstein (in German). *Ann. Physik* 1912;38:1056-1058. Cf. also the editors' comments in *The Collected Papers of Albert Einstein*. Princeton: Princeton University Press 1995;4:126.

[3] Foster J, Nightingale JD. *A Short Course in General Relativity*, 3rd edn. New York: Springer-Verlag 2006.

[4] Rossler OE, Kuypers H, Parisi J. Gravitational slowing-down of clocks implies proportional size increase. In: *A Perspective Look at Nonlinearr Media.* Springer *Lecture Notes in Physics* 1998;503:370-372.

[5] Rindler W. *Relativity – Special, General, and Cosmological*. Oxford: Oxford University Press 2001; p. 184.

[6] Greiner W. *Klassische Mechanik I, Kinematik und Dynamik der Punktteilchen, Relativität*. Frankfurt: Harri Deutsch 2003; p. 374. (Similarly in the edition of 1993.)

[7] Pound R, Rebka GS. Apparent weight of photons. *Phys. Rev. Lett.* 1960;4:337-341.

[8] Kuypers H. Atoms in the gravitational field: hints at a change of mass and size (in German). *Doctoral dissertation*, submitted to the Faculty of Chemistry and Pharmacy of the University of Tubingen, September 2005. (Referees: Rössler, Greiner, Neugebauer).

[9] Rossler OE, Kuypers H. The scale change of Einstein's equivalence principle. *Chaos, Solitons and Fractals* 2005;25:897-899.

[10] Finkelstein D. Past and future asymmetry of the gravitational field of general relativity. *Phys. Rev.* 1958;110:965-967.

[11] Shapiro II. Fourth test of general relativity. *Phys. Rev. Lett.* 1964; 13:789-791.

[12] Muhleman D, Richley P. Effects of general relativity on planetary radar distance measurements. *Space Programs Summary* 1964; 4 (No. 37-39):239-241.

[13] Bertotti B, Iess L, Tortora P. *Nature* 2003; 425:374

[14] Wald RM. *General Relativity*, Chicago: Chicago University Press 1984, p. 151 pp.

[15] Oppenheimer JR, Snyder H. On continued gravitational contraction. *Phys. Rev.* 1939;56:455-459.

[16] Rossler OE. Abraham-like return to constant c in general relativity: "$\mathfrak{R}$-theorem" demonstrated in Schwarzschild metric, www.wissensnavigator.com/documents/ottoroesslerminiblackhole.pdf (second of the two papers there)

[17] "Ich". (15.07.2008, 14:34), http://astronews.com/forum/showpost.php?p=39986&postcomt=504

[18] Hawking WS. Black hole explosions. *Nature* 1974; 248:30-31.

[19] "Ich". *Entfernungen in der Schwarzschildmetrik*, http://www.achtphasen.net/miniblackhole/Ich/Schwarzschild.pdf

[20] Horowitz GT, Maeda K. Fate of the black string instability. *Phys. Rev. Lett.* 2001;87: 1301301.

[21] Kleihaus B, Kunz J, Navarro-Lérida D. New dimensions for black holes: Black rings and black strings – exotic objects in more than threee spatial dimensions (in German). *Physik Journal* 2008; 7(No. 2):41-47.

[22] Thorne KS. *Black Holes and Time Warps*. New York: Norton 1994.

[23] Rossler OE, Kuypers K, Diebner HH, ElNaschie MS. Almost black holes: an old-new paradigm. *Chaos, Solitons and Fractals* 1998; 9:1025-1034.

[24] ElNaschie MS. Transfinite harmonization by taking the dissonance out of the quantum field symphony. *Chaos, Solitons and Fractals* 2008;36:781-786.

[25] Ahmed NM. Biography of Georg Cantor. *The Postgraduate Magazine* 2007;1:4-14 (School of Mathematics and Statistics, University of Newcastle Upon Tyne).

[26] O.E. Rossler, Abraham-solution to Schwarzschild metric implies that CERN miniblack holes pose a planetary risk,
http://www.wissensnavigator.com/documents/ottoroesslerminiblackhole.pdf
(first of the two papers there);
published (July 2008) in: *Vernetzte Wissenschaften – Crosslinks in Natural and Social Sciences*, Plath PJ, Hass EC, ed. Berlin: Logos Verlag 2009; p. 263-270.
(A publication of the *Flowing Institute* of the Leibniz Sozietät.)
ISBN 978-3-8325-1947-7

[27] Nicolai H. *Comments from Prof. Dr. Hermann Nicolai, Director, Max-Planck-Institut für Gravitationsphysik (Albert-Einstein-Institut) Potsdam, Germany on speculations raised by Professor Otto Roessler about the production of black holes at the LHC*, http://environmental-impact.web.cern.ch/environmental-impact/Objects/LHCSafety/NicolaiComment-en.pdf

[28] Giulini D, Nicolai H. *On the arguments of O.E. Rössler*, http://environmental-impact.web.cern.ch/environmental-impact/Objects/LHCSafety/NicolaiFurtherComment-en.pdf (July 30, 2008)

[29] *The earth will not be devoured by black holes. Statement given by KET on the assertions of Prof. Otto Rössler* (in German), August 2008,

www.ketweb.de/stellungnahmen/20080730_Stellungnahme_zu_den_Behauptungen_von_Prof_Otto_Roessler.pdf

[30] Rossler OE, *A Petition to CERN*, http://www.wissensnavigator.com/documents/PetitiontoCERN.pdf

[31] Pais A. *Einstein Lived Here*. New York: Oxford University Press 1994; p. 219.

[32] Szilard L. The Mark Gable Foundation (1948). In: *The Voice of the Dolphins*, Szilard L. ed. New York: Simon and Schuster 1961;
German edition: Szilard L. Die Mark-Gable-Stiftung. In: *Die Stimme der Delphine*, Science-fiction-Erzählungen, Vorwort C.F. von Weizsäcker, Vom Autor durchgesehene Fassung. Frankfurt am Main: Suhrkamp 1981; p. 37-90.

[33] Reiser M. "Why I am returning my chair" (in German), *Frankfurter Allgemeine Zeitung*, January 14, 2009, p. N 5 .

[34] Hawking SW, Hawking L. *George's Secret Key to the Universe*. New York: Simon and Schuster 2007;
Hawking SW, Hawking L. *George and the Cosmic Treasure Hunt*. London: Random House 2009. .

[35] Israel W. Gedanken experiments in black hole thermodynamics. In: *Black Holes: Theory and Observation.* Springer *Lecture Notes in Physics* 1998;514:339-363, p. 355.

[36] Marmet P. *Einstein's Theory of Relativity vs. Classical Mechanics.* Concester, Ontario, Canada: New Physics Boks 1997; chapter 1.7.
www.newtonphysics.on.ca/EINSTEIN/Chapter1.html

[37] Rossler OE. A rational and moral and spiritual dilemma, http://www.wissensnavigator.com/documents/spiritualottoeroessler.pdf;
published two months later (July 2008) in: *Personal and Spiritual Development in the World of Cultural Diversity*, Lasker GE, Hiwaki K, eds., Vol. 5. Tecumseh, Ontario, Canada: The International Institute for Advanced Studies in Systems Research and Cybernetics (IIAS) 2008; p. 61-66.
ISBN 978-1-897233-11-5

[38] Bell JS. How to teach special relativity. *Progress in Scientific Culture* 1976;1(No. 2); reprinted in: Bell JS. *Speakable and Unspeakable in Quantum Mechanics*. Cambridge: Cambridge University Press 1987; p. 67-80.

[39] Penrose R. *The Road to Reality – A Complete Guide to the Laws of the Universe.* London: Vintage Books 2004, p. 841 and Fig. 30.15

[40] *Jonas' World, The Thinking of a Child* (in German). Rossler R, Rossler OE, eds. Reinbek: Rowohlt 1994; p. 51. (Age 5,5)

Link for transcript for April 25 2010 Truth Radio interview:

http://www.bryanfarnum.ca/pdf/Interview_Dr._Rossler_University_Tubingen_re_Safety_Concerns__CERN_LHC.pdf